Texas Geometry Scope and Sequence, 2025-2026

Texas Essential Knowledge and Skills for Mathematics

Students began their study of geometric concepts in middle school mathematics. They studied area, surface area, and volume and informally investigated lines, angles, and triangles. Students in middle school also explored transformations, including translations, reflections, rotations, and dilations.

The Geometry course outlined in this document begins with developing the tools of geometry, including transformations, proof, and constructions. These tools are used throughout the course as students formalize geometric concepts studied in earlier courses and extend those ideas to new concepts presented in the high school standards. There is a focus on modeling, problem solving, and proof throughout the course.

Throughout this Geometry course, students use mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

These processes should become the natural way in which students come to understand and do mathematics. While, depending on the content to be understood or on the problem to be solved, any process might be seen or applied, some processes may prove more useful than others.

These course materials are designed to support 151-186 lessons of instruction and assessment (1 lesson equals 45 minutes).

Agile Mind Topics	Topic Descriptions	Texas Essential Knowledge and Skills for Mathematics • Standards listed in black are the primary instructional focus of the topic. • Standards in gray support topic content or indicate foundations for future work.
Geometric transformations (20-22 lessons)		
1: Using inductive reasoning and conjectures 8 lessons	This topic introduces students to inductive reasoning, the process of observing and forming conclusions about patterns. Using paper folding and inductive reasoning, students observe, discover, and analyze properties of angle bisectors and then develop two conjectures about them. This topic explores basic geometry terms and notations including point, line, plane, line segment, ray, angle, congruence, midpoint, and vertex. Complementary angles and supplementary angles are also reviewed. This topic provides opportunities for students to use the process standards as they communicate and describe geometric relationships that they see in the world around them using formal language and notation. Students use tools, including Patty Paper to investigate angle and segment bisectors, write conjectures, and justify their conclusions. Students represent bisectors in multiple ways using technology, diagrams, and paper-folding constructions.	**(1) Mathematical process standards:** (A), (C), (D), (F), (G) **(4) Logical argument and constructions.** The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to: (A) distinguish between undefined terms, definitions, ~~postulates,~~ conjectures, and ~~theorems~~ **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, including ~~angles formed by parallel lines cut by a transversal, criteria required for triangle congruence,~~ special segments of triangles, ~~diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles~~ choosing from a variety of tools (C) use the constructions of ~~congruent segments, congruent angles,~~ angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (A) ~~verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and~~ prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems **ELPS:** 1.A, 1.B, 1.D, 1.E, 1.F, 2.C, 2.E, 2.I, 3.B, 3.C, 3.D, 3.E, 3.F, 3.G, 3.J, 4.C, 4.E, 4.F, 4.G, 5.B, 5.F, 5.G

2: Rigid transformations 6 lessons **Corequisite support** 0-1 lesson **APPENDIX: Transformational geometry and similarity**	Rigid transformations of a geometric shape do not change length, area, or angle measure. This topic explores three basic rigid transformations: reflections, translations, and rotations. Students use these transformations to discover and prove geometric properties and to produce patterns, specifically tessellations. The activities in this topic support the students in engaging the process of selecting appropriate tools as they learn to use tools including Patty Paper and dynamic geometry technology to analyze geometric relationships. Students apply transformations to solve problems that arise in the workplace in the cable problem and implement the beginning states of the problem-solving routine to analyze the information and formulate a plan for determining the cable length. Students communicate about compositions of transformations and the geometric relationships that result.	**(1) Mathematical process standards:** (A), (B), (C), (D), (F), (G) **(3) Coordinate and transformational geometry.** The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to: (B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, ~~a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane~~ (C) identify the sequence of transformations that will carry a given pre-image onto an image ~~on and~~ off the coordinate plane (D) identify and distinguish between reflectional and rotational symmetry in a plane figure **ELPS:** 1.A, 1.E, 2.C, 2.E, 3.F, 3.J, 4.E, 5.G **Corequisite standards:** 8.10.A, 8.10.C
3: Transformations and coordinate geometry 6-8 lessons *(Lessons 6 and 7 are optional)*	This topic introduces coordinate geometry as a tool for discovering and verifying properties of geometric shapes, using ordered-pair rules to describe reflections, translations, and rotations of a figure. Students explore slope and distance, preservation of measurements, and collinearity and betweenness. The topic also compares rigid transformations to nonrigid transformations. Finally, the topic includes an optional extension that uses matrices to describe transformations.	**(1) Mathematical process standards:** (A), (C), (D), (E), (F), (G) **(2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:** (B) ~~derive and~~ use ~~the distance~~, slope, ~~and midpoint~~ formulas to verify geometric relationships, including ~~congruence of segments and~~ parallelism ~~or perpendicularity~~ of pairs of lines **(3) Coordinate and transformational geometry.** The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to: (A) describe and perform transformations of figures in a plane using coordinate notation

	In this topic, students use the process standards to apply transformations to real-world situations, including city planning, nature, and computer animation. Students use tools including the coordinate plane, MIRAs, technology, and Patty Paper to create representations of transformations. Students use ordered pairs to record and communicate precisely transformations on the coordinate plane. Students also use coordinates to make mathematical arguments showing parallel lines using transformations.	(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, ~~a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane~~ (C) identify the sequence of transformations that will carry a given pre-image onto an image on ~~and off~~ the coordinate plane (6) **Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (A) verify theorems about ~~angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and prove~~ equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems **ELPS:** 1.E, 2.C, 2.E, 2.I, 3.C, 3.D, 3.E, 3.F, 4.F
Deductive reasoning with angles and lines (17-18 lessons)		
Corequisite support 0-4 lessons **APPENDIX: Solidifying your skills with functions and equations** **APPENDIX: Operations on polynomials**	In the *Exploring* “Consolidating your skills with quadratic functions and equations” from the topic **Solidifying your skills with functions and equations**, students can review key characteristics of linear and quadratic functions. Students review their skills with solving linear equations. They also review and strengthen their fluency with quadratic functions and equations as they create and analyze graphs of quadratic functions, multiply and factor polynomial expressions, and solve quadratic equations using graphs, factoring, and the quadratic formula. The indicated lessons from the topic **Operations on polynomials** explores polynomial operations. Students learn how to multiply, add, and subtract polynomials using concrete models and analytic	*Corequisite standards:* **(8.8) Expressions, equations, and relationships.** The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to: (C) model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants **(A1.10) Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to: (A) add and subtract polynomials of degree one and degree two (B) multiply polynomials of degree one and degree two (D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property (E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two

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	techniques. They also learn how to factor trinomials using concrete models and analytic techniques.	
4: Deductive reasoning, logic, and proof 6 lessons **Corequisite support** 0-4 lesson **APPENDIX: Solving quadratic equations** **APPENDIX: Solidifying your skills with functions and equations**	In this topic, students learn how to combine true statements within a mathematical system to deductively prove that some other statement is true for that system. As they explore this topic, students complete flow-chart and two-column proofs, including a proof of the Vertical Angle Theorem. Students compare and contrast multiple proofs of a statement, including transformational and axiomatic approaches. They learn that, once a statement is proven, it can be used to help prove additional statements. As students learn about formal proof in this topic, they have multiple opportunities apply the process standards as they communicate mathematical ideas and reasoning. They are introduced to multiple formats of proof and represent their reasoning in a variety of ways. This topic sets the foundation for students' work with justifying mathematical ideas using precise language that will be used throughout the course.	**(1) Mathematical process standards:** (D), (E), (G) (3) **Coordinate and transformational geometry.** The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to: (C) identify the sequence of transformations that will carry a given pre-image onto an image ~~on and~~ off the coordinate plane **(4) Logical argument and constructions.** The student uses the process skills with deductive reasoning to understand geometric relationships. Student is expected to: (A) distinguish between ~~undefined terms, definitions~~, postulates, conjectures, and theorems **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, ~~and angles formed by parallel lines cut by a transversal and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems~~ **ELPS:** 2.F, 2.I, 3.G, 3.H, 3.J, 4.F, 4.G, 5.F, 5.G **Corequisite standards:** A1.8.A
5: Conditional statements 6-7 lessons *(Lesson 6 is optional)*	This topic introduces the conditional or if-then statement and demonstrates how to represent if-then statements using both logic notation and Euler diagrams. It allows students to explore the application of formal logic rules to the statement of the converse of a conditional. Students learn how to write, notate and diagram the inverse and	**(1) Mathematical process standards:** (D), (E), (F), (G) **(4) Logical argument and constructions.** The student uses the process skills with deductive reasoning to understand geometric relationships. Student is expected to: (B) identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse

	contrapositive of a conditional and explore connections to biconditional statements. It also introduces students to the technique of indirect proof, which relies on the consequences of logical contradictions. Students use the process of communicating mathematical ideas and their implications as they make sense of conditional statements. They use precise language, Euler diagrams, and logic notation to represent conditional statements and display their reasoning.	(C) verify that a conjecture is false using a counterexample **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, ~~and angles formed by parallel lines cut by a~~ transversal~~ and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems~~ **ELPS:** 1.E, 2.C, 3.E, 3.F, 4.D, 4.E, 5.B, 5.F
6: Lines and transversals 5 lessons **Corequisite support** 0-1 lesson **APPENDIX: Solidifying your skills with functions and equations**	This topic explores lines, transversals, and special angles associated with them. Students distinguish between parallel and skew lines. They use transformations to explore properties of corresponding angles, and then related properties of alternate interior and exterior angles and consecutive interior angles, formed when parallel lines are cut by a transversal. Students also learn how to use angle congruence to establish that two lines are parallel. In this topic, students apply the process standards as they identify lines in everyday objects. Students make use of tools including paper and pencil and dynamic geometry technology to organize and record their thinking about the relationships they explore. They justify their conjectures about the angles formed by parallel lines and write conclusion about the various angle pairs.	**(1) Mathematical process standards:** (A), (C), (D), (E), (G) **(2) Coordinate and transformational geometry.** The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to: (B) derive and use the ~~distance~~, slope, ~~and midpoint~~ formulas to verify geometric relationships, including ~~congruence of segments~~ and parallelism or perpendicularity of pairs of lines (C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, ~~criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools~~ **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (A) verify theorems about ang~~les formed by the intersection of lines and line segments, including vertical angles, and~~ angles formed by parallel lines cut by a transversal ~~and prove equidistance between the endpoints of a segment and~~

		~~points on its perpendicular bisector~~ and apply these relationships to solve problems **ELPS:** 2.D, 2.E, 3.E, 4.F, 4.G, 5.B, 5.F, 5.G **Corequisite standards:** A1.2.B, A1.2.C, A1.3.A, A1.3.C
Triangles (9-12 lessons)		
7: Properties of a triangle 6 lessons	This topic explores basic theorems and conjectures about triangles, including the triangle inequality conjecture and the Triangle Sum Theorem. Students also investigate the Exterior Angle Theorem and make conjectures about isosceles triangles. Students use transformations to verify properties of isosceles triangles. Students learn about the rigid nature of triangles and the importance of rigidity in construction. Students apply the process standards as they organize their ideas about triangles using a concept map. They analyze mathematical relationships related to triangles using a variety of tools, including technology and Patty Paper. They write conjectures and prove them using flow-chart proofs, two-column proofs, and paragraph proofs. Students also make connections between geometric relationships and algebraic processes to solve triangle problems.	**(1) Mathematical process standards:** (A), (C), (D), (E), (F), (G) **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, including ~~angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals,~~ interior and exterior angles of polygons, a~~nd special segments and angles of circles~~ choosing from a variety of tools (D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles (D) verify theorems about relationships in triangles, including ~~proof of Pythagorean Theorem,~~ the sum of interior angles, base angles of isosceles triangles, ~~midsegments, and medians,~~ and apply these relationships to solve problems **ELPS:** 1.A, 1.E, 2.C, 2.D, 2.E, 2.I, 3.D, 3.J, 4.F
8: Special lines and points in triangles 3-6 lessons *(Lessons 2, 3 and 5 are optional)*	Some of the greatest discoveries in mathematics were made because someone took a simple idea, pattern, or concept and studied it deeply. This topic invites students to experience this joy of exploration by facilitating a deep study of angle bisectors, perpendicular bisectors, medians, and altitudes leading to some special points on a triangle—the incenter, the circumcenter, the	**(1) Mathematical process standards:** (A), (B), (C), (E), (F) **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, ~~including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence~~, special segments of triangles~~, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles~~ choosing from a variety of tools

	centroid, and the orthocenter—and how they are related to one another. In this topic, students apply the process standards as they make connections between equations of lines, systems of equations, and centers of a triangle. Students connect the centroid of a triangle to the center of gravity of the shape and solve a problem involving dividing property. Students explore measurements on a triangle and organize their data in a table, then make conjectures based on the data. Students engage in a gallery walk, sharing their solution strategies, checking their solutions for reasonableness and refining their solutions.	(C) use the constructions of ~~congruent segments, congruent angles,~~ angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (D) verify theorems about the relationships in triangles, including ~~proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and~~ medians, and apply these relationships to solve problems **ELPS:** 1.A, 1.E, 1.F, 2.C, 2.E, 2.I, 3.B, 3.D, 3.F, 3.G, 4.C, 4.F, 4.G, 5.G
Triangle congruence (15–19 lessons)		
9: Congruent triangle postulates 6 lessons	This topic focuses on shortcuts to prove two triangles congruent. Students also learn how to use notation to correctly communicate correspondence between sides and angles of congruent triangles. In this topic, students apply the process skills as they investigate the congruent triangle postulates. They engage in the problem-solving routine as they try out various shortcuts to proving triangles congruent and	**(1) Mathematical process standards:** (B), (C), (D), (E), (F), (G) **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, including ~~angles formed by parallel lines cut by a transversal,~~ criteria required for triangle congruence, ~~special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles~~ choosing from a variety of tools **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as

	select between technology and straws to explore the triangles. They must analyze the triangles given various conditions and communicate about their reasoning as they determine if each shortcut is valid or not. As they complete their explorations, they organize their conjectures in a summary chart.	coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions (C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles' **ELPS:** 2.I, 3.D, 3.F, 3.J, 4.F, 5.B, 5.F, 5.G
10: Using congruent triangles 4-7 lessons *(Lessons 4, 5, and 6 are optional)*	In this topic, students learn to apply the fact that corresponding parts of congruent triangles are congruent. Students use congruent triangles to prove the isosceles triangle conjectures made earlier, and they learn to pull apart complex diagrams to find congruent triangles. Students also explore the Hinge Theorem. Students use the process of justifying and applying geometric theorems by proving triangle congruence with side-angle-side, angle-side-angle, side-side-side, angle-angle-side, and hypotenuse-leg. Students identify and verify congruence through rigid transformations that help to recognize congruent figures and their corresponding sides and angles to solve geometric problems.	**(1) Mathematical process standards:** (A), (E), (G) **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions (C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles (D) verify theorems about the relationships in triangles, including ~~proof of the Pythagorean Theorem,~~ the sum of interior angles, base angles of isosceles triangles, ~~midsegments, and medians,~~ and apply relationships to solve problems **ELPS:** 1.E, 2.I, 3.C, 3.D, 3.E, 3.F, 4.G, 5.B, 5.F
11: Compass and straightedge constructions 5-6 lessons *(Lesson 3 is optional)*	This topic introduces the principles important in creating and analyzing formal geometric constructions using a compass and a straightedge. Because the steps for making a construction, along with their justifications, function like the statements and reasons in a proof, the result of the construction is true in the same way that the conclusion of a deductive proof must be true. Students engage in the process standards as they learn to use a new tool for	**(1) Mathematical process standards:** (C), (F), (G) **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge (C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships

	constructions, the compass and straightedge. Students use precise language to differentiate between a construction and a sketch or drawing. Students also justify statements, including constructions, equidistance between segment endpoints and points on a perpendicular bisector and demonstrate triangle congruence using congruence theorems.	**(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (A) ~~verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and~~ prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems (B) prove two triangles are congruent by applying the Side-Angle-Side, ~~Angle-Side-Angle~~, Side-Side-Side, Angle-Angle-Side, and ~~Hypotenuse-Leg~~ congruence conditions **ELPS:** 1.B, 1.F, 2.C, 2.F, 2.I, 3.B, 3.G, 3.H, 3.J, 5.G
Similarity transformations (9-10 lessons)		
12: Dilations and similarity 9-10 lessons *(Lesson 3 is optional)* **Corequisite support** 0-1 lesson **APPENDIX: Transformational geometry and similarity** **APPENDIX: Solidifying your skills with numbers**	Focusing on dilations and similarity, the topic begins by defining these terms mathematically, including the concepts of center point and scale factor. Students learn to produce an accurate dilation of a geometric shape by a specific scale factor. Students also study dilations whose center is not at the origin of a coordinate plane and connect to partitioning a segment, including finding the midpoint. Students analyze dilations of lines and circles. Students learn how to prove two triangles are similar and that all circles are similar to each other. Students apply their knowledge of similarity to explore proportions in parallel lines and geometric mean. Midsegments are approached as a special case of the proportions involving parallel lines. Students apply these relationships in real world problem situations. Students use the process of constructing logical arguments to make and validate geometric conjectures, apply similarity	**(1) Mathematical process standards:** (A), (B), (C), (D), (F), (G) **(2) Coordinate and transformational geometry.** The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to: (A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint **(3) Coordinate and transformational geometry.** The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to: (A) describe and perform transformations of figures in a plane using coordinate notation (B) determine the image or pre-image of a given two-dimensional figure under ~~a composition of rigid transformations~~, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane (C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane

	criteria such as Angle-Angle, and use the Triangle Proportionality theorem to solve problems involving similar figures. Students investigate patterns and prove theorems, such as those involving midsegments in triangles and similarity criteria through coordinate and transformational proofs.	**(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, including ~~angles formed by parallel lines cut by a transversal,~~ ~~criteria required for triangle congruence,~~ special segments of triangles, ~~diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles~~ choosing from a variety of tools **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (D) verify theorems about the relationships in triangles, including ~~proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles,~~ midsegments, ~~and medians,~~ and apply relationships to solve problems **(7) Similarity, proof, and trigonometry.** The student uses the process skills in applying similarity to solve problems. The student is expected to: (A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles (B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems **(8) Similarity, proof, and trigonometry.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (A) prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems (B) identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems **ELPS:** 1.A, 1.F, 2.C, 3.F, 3.J, 4.D, 4.F, 5.F, 5.G **Corequisite standards:** 8.3.A, 8.3.B, A1.11.A

Right triangle relationships and trigonometry (12-13 lessons)		
13: Pythagorean Theorem and the distance formula 5-6 lessons *(Lesson 2 is optional)* **Corequisite support** 0-1 lesson **APPENDIX: Pythagorean Theorem**	This topic explores proofs of the Pythagorean Theorem, using concrete models and algebraic representations. In proving the converse of the Pythagorean Theorem, students are guided to develop related conjectures about acute triangles and obtuse triangles. Students then solve problems using the Pythagorean Theorem, including problems in the coordinate plane. Students develop the distance formula and midpoint formula. Students apply the process of analyzing relationships as they examine the coordinate systems to find fractional distances, midpoint, and apply the formulas for distance, midpoint, and slope to verify segment congruence. Students also employ deductive reasoning to validate geometric relationships, test converses, and use counterexamples. Students engage with various proof methods and use them to solve problems connecting geometric concepts to strengthen problem solving skills and solve real-world problems	**(1) Mathematical process standards:** (A), (B), (D), (F), (G) **(2) Coordinate and transformational geometry.** The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to: (A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint (B) derive and use the distance, ~~slope~~, and midpoint formulas to verify geometric relationships, including congruence of segments ~~and parallelism or perpendicularity of pairs of lines~~ **(4) Logical argument and constructions.** The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to: (B) identify and determine the validity of the converse, ~~inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse~~ © verify that a conjecture is false using a counterexample **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to: (D) verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, ~~the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and~~ apply these relationships to solve problems **(9) Similarity, proof, and trigonometry.** The student uses the process skills to understand and apply relationships in right triangles. The student is expected to: (B) apply the relationships in ~~special~~ right triangles ~~30°‑60°‑90° and 45°‑45°‑90°~~ and the Pythagorean theorem, including Pythagorean triples, to solve problems **ELPS:** 1.A, 1.D, 3.B, 3.C, 3.D, 3.G, 3.H, 4.C, 4.D, 4.F, 5.B, 5.G **Corequisite standards:** 8.6.C, 8.7.C

14: Right triangle and trig relationships 7 lessons	This topic extends the idea of triangle similarity to indirect measurements. Students develop properties of special right triangles, and use properties of similar triangles to develop their understanding of trigonometric ratios. These ideas are then applied to find unknown lengths and angle measurements. Students engage the process of selecting tools, including rulers, protractors, Patty Paper, and a sextant, as they explore right triangles and trigonometric relationships. They Students justify the properties of special right triangles and solve problems connecting geometric concepts with trigonometric applications.	**(1) Mathematical process standards:** (A), (B), (C), (D), (F), (G) **(9) Similarity, proof, and trigonometry.** The student uses the process skills to understand and apply relationships in right triangles. The student is expected to: (A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems (B) apply the relationships in special right triangles 30°- 60°- 90° and 45°- 45°- 90° ~~and the Pythagorean theorem, including Pythagorean triples,~~ to solve problems **ELPS:** 1.B, 1.D, 1.E, 1.F, 2.C, 2.D, 2.F, 2.I, 3.B, 3.C, 3.D, 3.E, 3.G, 3.H, 3.J, 4.C, 4.D, 4.F, 4.G
Quadrilaterals and other polygons (7 lessons)		
15: Polygons and special quadrilaterals 7 lessons	In this topic, students learn about the parts and characteristics of polygons, including quadrilaterals. They classify polygons according to the number of sides and whether they are convex or concave, and they apply what they know about triangles to investigate measures of interior and exterior angles of polygons. Students learn to classify quadrilaterals by exploring their properties. They apply the properties of different quadrilaterals to construct transformational and coordinate proofs. Students use the process of justifying mathematical arguments as they applying distance, slope, and midpoint formulas with one- and two-dimensional coordinate systems to corroborate geometric relationships. Students use these formulas to prove relationships through logical constructions, investigate patterns in figures	**(1) Mathematical process standards:** (C), (D), (E), (F), (G) **(2) Coordinate and transformational geometry.** The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to: (B) ~~derive and~~ use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, including ~~angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles,~~ diagonals of quadrilaterals, interior and exterior angles of polygons, ~~and special segments and angles of circles~~ choosing from a variety of tools **(6) Proof and congruence.** The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

	like quadrilaterals and polygons, and utilize various tools to form and test conjectures.	(E) prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships to solve problems **ELPS:** 1.A, 1.E, 2.E, 2.I, 3.E, 3.G, 3.J, 4.F, 5.B, 5.F, 5.G
Circles (17-19 lessons)		
16: Algebraic representations of circles 4-6 lessons *(Lessons 4 and 5 are optional)*	Students review the basic characteristics of a circle as they derive and apply equations of circles. An optional extension is included in this topic dealing with curves of constant width, explored through circles and Reuleaux triangles. Students apply the process of analyzing mathematical relationships and making connections as they derive the equation of a circle. Students apply the equation of a circle to model real-world situations and connect geometric properties with algebraic manipulations to communicate their reasoning.	**(1) Mathematical process standards:** (A), (D), (F), (G) **(12) Circles.** The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to: (E) show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k), $(x - h)^2 + (y - k)^2 = r^2$ **ELPS:** 1.A, 1.B, 2.E, 2.F, 4.C, 4.F, 4.G, 5.G
17: Chords, arcs, and inscribed angles 7 lessons	This topic examines special relationships among chords, arcs, and angles in a circle. Relationships between chords and radii and between chords and their intercepted arcs are explored. The relationship between the measures of inscribed angles and their intercepted arcs is also discussed, as are the various relationships among the measures of angles and arcs formed when two lines intersect a circle in various ways. Students use the process of validating geometric conjectures and applying theorems to communicate mathematical ideas. Students investigate geometric relationships using constructions to explore patterns, explore theorems related to radii,	**(1) Mathematical process standards:** (A), (C), (F), (G) **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, including ~~angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and~~ special segments and angles of circles choosing from a variety of tools **(12) Circles.** The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to: (A) apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems **ELPS:** 1.E, 1.F, 2.D, 3.B, 3.H, 4.C, 4.F, 5.B, 5.F, 5.G

	chords, tangents, and secants to solve abstract problems.	
18: Lines and segments on circles 6 lessons	This topic focuses on properties of chords, tangents, and secants on circles. Among the tangent line properties discussed is the relationship between a tangent and a radius. Connections are also made between tangents, secants, and the lengths of related segments that they form with regard to a circle. Students use the process of validating geometric conjectures and applying theorems to communicate mathematical ideas. Students investigate geometric relationships using constructions to explore patterns, explore theorems related to radii, chords, tangents, and secants to solve abstract and real-world problems.	**(1) Mathematical process standards:** (A), (B), (C), (F), (G) **(5) Logical argument and constructions.** The student uses constructions to validate conjectures about geometric figures. The student is expected to: (A) investigate patterns to make conjectures about geometric relationships, ~~including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and~~ special segments and angles of circles choosing from a variety of tools **(12) Circles.** The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to: (A) apply theorems about circles, including relationships among ~~angles,~~ radii, chords, tangents, and secants, to solve non-contextual problems **ELPS:** 1.A, 1.E, 1.F, 2.E, 3.E, 3.G, 4.G, 5.B, 5.F, 5.G
Geometric modeling in two dimensions (12-14 lessons)		
19: Modeling with area 6-7 lessons *(Lesson 6 is optional)*	This topic discusses the basic concept of area. It investigates familiar mathematical formulas through shearing figures between two parallel lines, and slicing figures and rearranging the pieces. The idea of using known areas of familiar shapes to estimate areas of unfamiliar, irregular regions is explored. Students use the process of mathematical reasoning to apply formulas for the area of regular polygons and composite shapes, such as triangles, parallelograms, and sectors of circles. Students decompose complex figures into simpler components, calculating the area appropriately using formulas accurately. Students solve real-world problems related to water flow in a river.	**(1) Mathematical process standards:** (A), (B), (C), (E), (F), (G) **(11) Two-dimensional and three-dimensional figures.** The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to: (A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure (B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, ~~or sectors of circles~~ to solve problems using appropriate units of measure **ELPS:** 1.B, 1.E, 2.D, 2.I, 3.C, 3.D, 3.E, 3.G, 3.H, 4.E,

20: Arc length and sectors of circles 6-7 lessons *(Lesson 3 is optional)*	In this topic, students revisit formulas for circumference and area of circles with a focus on proportionality. Students examine arc length and radian measures. They also develop the familiar formula for area of a circle by relating the circle to approximating parallelograms. Related formulas for areas of sectors and segments of circles are presented, and connections between sectors and cones are explored Students apply the process standards as they analyze the proportional relationship between the measure of arc length and the circumference of a circle. Students use precise language to define radian measure based on the ratio of arc length and the ratio of the circle.	**(1) Mathematical process standards:** (A), (B), (C), (D), (F), (G) **(12) Circles.** The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to: (B) apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems (C) apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems (D) describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle **(13) Probability.** The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to: (B) determine probabilities based on area to solve contextual problems **ELPS:** 1.A, 1.D, 2.D, 2.E, 3.E, 4.D, 4.E, 4.F, 4.G
Understanding and modeling with three-dimensional figures (19-28 lessons)		
21: Relating 2-D and 3-D objects 3-6 lessons *(Lessons 1, 2, and 5 are optional)*	Making two-dimensional representations of the three-dimensional world requires skill. In this topic, students learn to represent three-dimensional objects using two-dimensional techniques, including nets, plane and cross sections, and solids of revolution. An optional block on orthographic and isometric drawings is also included. As students explore these different representations, students deepen their understanding of the three-dimensional objects they represent. Students use process skills to identify the shape of two-dimensional cross sections of prisms, pyramids, cylinders, cones, and spheres in abstract figures and rea objects. Students analyze three dimensional objects generated by rotating two-dimensional shapes using technology.	**(1) Mathematical process standards:** (A), (C), (F) **(10) Two-dimensional and three-dimensional figures.** The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to: (A) identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two- dimensional shapes **ELPS:** 1.D, 1.E, 2.E, 2.I, 3.B, 3.E, 4.F

22: Prisms and cylinders 5-6 lessons *(Lesson 4 is optional)*	This topic explores prisms and cylinders. Students learn basic terminology and develop basic formulas for volume and surface area and see how these formulas generalize across different kinds of prisms. Students engage the process standards as they solve real-world problems involving prisms and cylinders as they analyze the reasonableness of their solutions in the swimming pool problem. They formulate strategies to accurately evaluate problems involving formulas for the total and lateral surface area. Students solve these problems by accurately calculating these measures using appropriate units.	**(1) Mathematical process standards:** (A), (B), (C), (D) **(10) Two-dimensional and three-dimensional figures.** The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to: (A) identify the shapes of two-dimensional cross-sections of prisms, ~~pyramids~~, cylinders, ~~cones, and spheres and identify three-dimensional objects generated by rotations of two- dimensional shapes~~ **(11) Two-dimensional and three-dimensional figures.** The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to: (C) apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, ~~pyramids, cones,~~ cylinders, ~~spheres, and composite figures,~~ to solve problems using appropriate units of measure (D) apply the formulas for the volume of three-dimensional figures, including prisms, ~~pyramids, cones,~~ cylinders, ~~spheres, and composite figures,~~ to solve problems using appropriate units of measure **ELPS:** 1.A, 2.E, 2.F, 3.E, 3.G, 3.J, 4.F, 4.G
23: Pyramids and cones 4-5 lessons *(Lesson 4 is optional)*	In this topic, students investigate pyramids and cones. They develop strategies and formulas for computing volume and surface area, and see how these are connected not only to each other but also to the work students have already done with cylinders and prisms. In this topic, students apply the process standards as they make connections between the volume formulas of pyramids, cones, cylinders, and prisms. Students use the problem-solving routing as they formulate strategies to accurately evaluate problems involving formulas for the total and lateral surface area. Students solve these problems by accurately calculating these measures using appropriate units.	**(1) Mathematical process standards:** (B), (C), (D), (F) **(10) Two-dimensional and three-dimensional figures.** The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to: (A) identify the shapes of two-dimensional cross-sections of ~~prisms,~~ pyramids, ~~cylinders,~~ cones, ~~and spheres and identify three-dimensional objects generated by rotations of two- dimensional shapes~~ **(11) Two-dimensional and three-dimensional figures.** The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to: (C) apply the formulas for the total and lateral surface area of three-dimensional figures, including ~~prisms~~, pyramids, cones, ~~cylinders, spheres, and composite figures,~~ to solve problems using appropriate units of measure **(D) apply the formulas for the volume of three-dimensional figures, including ~~prisms,~~ pyramids, cones, ~~cylinders, spheres, and composite figures,~~ to solve problems using appropriate units of measure** **ELPS:** 1.A, 1.B, 1.E, 1.F, 2.E, 3.C, 3.D, 3.E, 3.H, 4.C, 4.F

24: Spheres 3-6 lessons *(Lessons 3, 4, and 5 are optional)*	This topic presents the basic characteristics and terminology associated with spheres. Students investigate strategies and formulas to compute volumes and surface areas and make connections among the formulas for cylinders, cones, and spheres. Students also learn about the surface area/volume "efficiency" of the sphere when compared to other three-dimensional shapes. Students engage in the process standards as they explore a justification of the formulas for surface area and volume of spheres. They solve real-world and abstract problems involving spheres and use graphs to analyze relationships.	**(1) Mathematical process standards:** (A), (C), (D), (F), (G) (10) **Two-dimensional and three-dimensional figures.** The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to: (A) identify the shapes of two-dimensional cross-sections of ~~prisms, pyramids,~~ cylinders, cones, and-spheres and identify three-dimensional objects generated by rotations of two- dimensional shapes **(11) Two-dimensional and three-dimensional figures.** The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to: (C) apply the formulas for the total and lateral surface area of three-dimensional figures, including ~~prisms, pyramids,~~ cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure **(D) apply the formulas for the volume of three-dimensional figures, including ~~prisms, pyramids,~~ cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure** **ELPS:** 1.A, 1.F, 2.C, 2.E, 2.F, 3.E, 4.F, 4.G
25: Analyzing dimensional changes 4-5 lessons *(Lesson 3 is optional)*	In this topic, students explore the effect of altering dimensions on the surface area and volume of a three-dimensional figure. This exploration includes both similar figures and non-similar solids. Students also see how comparing certain surface areas to volumes leads to a new kind of function—a rational function. Students use the process of employing problem-solving models and appropriate tools to understand the characteristics and dimensional change of two- and three-dimensional figures. Students also analyze changes that affect perimeter, area, surface area, and volume.	**(1) Mathematical process standards:** (A), (C), (D), (E), (F), (G) **(10) Two-dimensional and three-dimensional figures**. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to: (B) **determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change** **ELPS:** 1.F, 2.E, 3.D, 3.E, 4.F, 4.G, 5.B, 5.G

Non-Euclidean Geometry (2-5 lessons)		
26: Non-Euclidean Geometry 2-5 lessons *(Lessons 1-3 are optional)*	Euclidean Geometry is the focus of most high school geometry courses. By accepting certain terms, definitions, and postulates, Euclidean Geometry can be applied to understand the world. What if a different set of terms, definitions, and postulates were accepted as a starting point? This topic begins to explore other geometries. Students use the process of analyzing and comparing geometric properties and synthesizing information to construct logical arguments. Students also integrate problem solving skills to deepen their understanding of Euclidean and spherical geometries.	**(1) Mathematical process standards:** (A), (B), (C), (F) **(4) Logical argument and constructions.** The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to: (D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle **ELPS**: 1.A, 1.E, 1.F, 2.D, 2.E, 2.I, 3.B, 3.C, 3.E, 3.G, 3.J
Probability (12 lessons)		
27: Revisiting probability 7 lessons)	This topic develops some of the basic tools and terminology of probability, including the Fundamental Counting Principle. Students apply the addition rule and multiplication rule in solving problems. They calculate permutations and combinations in the service of answering probability questions. Students apply the process standards as they apply probability to real-world situations. They use manipulatives and various representations, including Venn diagrams, tables, and tree diagrams, to organize their thinking and make sense of problems. They use precise mathematical language to describe probability of events.	**(1) Mathematical process standards:** (A), (C), (D), (E), (F) **(13) Probability.** The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to: (A) develop strategies to use permutations and combinations to solve contextual problems **ELPS:** 1.E, 2.C, 2.E, 2.I, 3.B, 3.E, 4.F, 4.G, 5.B

28: Conditional probability and independence 5 lessons	In this topic, students build on the tools of probability developed in the previous topic to analyze conditional probability and independence. Students develop and apply rule for conditional probability and independence as they solve problems. Students use the process of selecting appropriate tools and techniques to communicate ideas, analyze relationships, identify independent events, and apply conditional probability. They justify solutions involving the independence and dependence of events using probability concepts.	**(1) Mathematical process standards:** (A), (D), (E), (F) **(13) Probability.** The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to: (C) identify whether two events are independent and compute the probability of the two events occurring together with or without replacement (D) apply conditional probability in contextual problems (E) apply independence in contextual problems **ELPS:** 1.D, 1.E, 2.I, 3.C, 3.E, 3.G, 3.H, 4.F, 5.G

APPENDIX: Key learning from earlier grades

The topics in this section provide support for key skills from earlier grades and co-requisite skills students may need to be successful with concepts in this course. We have provided this set of lessons and problem-solving resources that can be used for differentiated practice and review. Specific guidance on how to use these topics is provided in the accompanying co-requisite guide, however, teachers may choose to use these topics in the way that serves their students best. Teachers may choose to assign these resources to students for independent review and practice, or they may choose to use them in facilitating small-group instruction.

Agile Mind Topics	Topic Descriptions	Texas Essential Knowledge and Skills for Mathematics • Standards listed in black are the primary instructional focus of the topic. • Standards in gray support topic content or indicate foundations for future work.
Solidifying your skills with numbers	In this topic, students can review and strengthen their fluency with rational number operations as they work with positive and negative integers, decimals, and fractions. Students also review irrational numbers and simplify square roots. In addition to paper-and-pencil and online tasks, students engage with simulations and interactive animations that provide thousands of opportunities to build their knowledge and skills.	**(5.3) Number and operations.** The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to: (K) add and subtract positive rational numbers fluently; and **(6.3) Number and operations**. The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to: (A) classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers; (D) add, subtract, multiply, and divide integers fluently; and (E) extend representations for division to include fraction notation such as a/b represents the same number as a ÷ b where b ≠ 0. **(7.3) Number and operations.** The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions. The student is expected to: (A) add, subtract, multiply, and divide rational numbers fluently; and **(8.2) Number and operations.** The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to: (A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers; **(A1.11) Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to: (A) simplify numerical radical expressions involving square roots; and

Solidifying your skills with functions and equations	In this topic, students can review key characteristics of linear and quadratic functions. They can strengthen their fluency with writing and graphing equations of lines in slope-intercept and point-slope form as well as solving single-step and multi-step linear equations with rational coefficients. Students review and strengthen their fluency with quadratic functions and equations as they create and analyze graphs of quadratic functions, multiply and factor polynomial expressions, and solve quadratic equations using graphs, factoring, and the quadratic formula. In addition to paper-and-pencil and online resources, students engage with simulations and interactive animations that provide a review of foundational concepts.	**(8.8) Expressions, equations, and relationships.** The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to: (C) model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants; and **(A1.2) Linear functions, equations, and inequalities. The** student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to: (B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points; (C) write linear equations in two variables given a table of values, a graph, and a verbal description; **(A1.3) Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to: (A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$; (C) graph linear functions on the coordinate plane and identify key features, including *x*- intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems; **(A1.6) Quadratic functions and equations.** The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to: (B) write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$); and **(A1.7) Quadratic functions and equations.** The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to: (A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry; (B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions; and (C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

		(A1.8) Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to: (A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula; and **(A1.10) Number and algebraic methods**. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to: (B) multiply polynomials of degree one and degree two; (E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two; and
Transformational geometry and similarity	This topic introduces coordinate geometry as a tool for exploring transformations. Using ordered pairs and algebraic rules to describe reflections, translations, rotations, and dilations, students become more adept at solving problems in the coordinate plane. The work with congruence and similarity in this topic provides a foundation for the development of the formal definition of slope later in the course.	**(8.3) Proportionality.** The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to: (A) generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation ***Supporting Standard*** (B) compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane ***Supporting Standard*** (C) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation ***Readiness Standard*** **(8.8) Expressions, equations, and relationships.** The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to: (D) use informal arguments to establish facts about ~~the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and~~ the angle- angle criterion for similarity of triangles ***Supporting Standard*** **(8.10) Two-dimensional shapes.** The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to: (A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane ***Supporting Standard*** (B) differentiate between transformations that preserve congruence and those that do not ***Supporting Standard***

		(C) explain the effect of translations, reflections over the ***x***- or ***y***-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation ***Readiness Standard***
Pythagorean Theorem	This topic explores proofs of the Pythagorean Theorem and its converse, using concrete models and algebraic representations. Students then solve real-world problems using the Pythagorean Theorem and its converse. Students also apply the Pythagorean Theorem to calculate distance between two points in the coordinate plane.	**(8.6) Expressions, equations, and relationships.** The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to: (C) use models and diagrams to explain the Pythagorean theorem ***Supporting Standard*** **(8.7) Expressions, equations, and relationships.** The student applies mathematical process standards to use geometry to solve problems. The student is expected to: (C) use the Pythagorean Theorem and its converse to solve problems ***Readiness Standard*** (D) determine the distance between two points on a coordinate plane using the Pythagorean Theorem ***Supporting Standard***
Operations on polynomials	This topic explores polynomial operations. Students learn how to multiply, add, and subtract polynomials using concrete models and analytic techniques. They also learn how to factor trinomials using concrete models and analytic techniques. Finally, students divide polynomials and connect polynomial division to the concept of a rational expression, laying the foundation for work with the arithmetic of polynomial expressions in later courses.	**(10) Number and algebraic methods.** The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to: (A) add and subtract polynomials of degree one and degree two ***Supporting Standard*** (B) multiply polynomials of degree one and degree two ***Supporting Standard*** (C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend ***Supporting Standard*** (D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property ***Supporting Standard*** (E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two ***Readiness Standard*** (F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial ***Supporting Standard***

Solving quadratic equations	This topic focuses on solving quadratic equations that arise from quadratic functions. Students learn to solve these equations by graphing, factoring, using square roots, and completing the square, and they see how the solution methods are connected as they connect the roots of an equation, the ***x***-intercepts of a graph, and the zeros of a function.	**(6) Quadratic functions and equations.** The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to: (C) write quadratic functions when given real solutions and graphs of their related equations ***Supporting Standard*** **(7) Quadratic functions and equations.** Student applies mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, solutions to equations. The student is expected to**:** (B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions ***Supporting Standard*** **(8) Quadratic functions and equations.** The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to: (A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, ~~and applying the quadratic formula~~ ***Readiness Standard***